# EXHIBIT B

# Expert Report of Dr. Jonathan Mattingly

Pursuant to 28 U.S.C. § 1746, I, Jonathan Mattingly, declare as follows:

**<u>Qualifications</u>**

I am the Kimberly J. Jenkins Distinguished University Professor of New Technologies in the Mathematics Department at Duke University. I am also a Professor of Statistical Science at Duke. I was the Chair of the Duke Department of Mathematics between 2016 and 2020. My degrees are from the North Carolina School of Science and Math (High School Diploma), Yale University (B.S.), and Princeton University (Ph.D.). I grew up in Charlotte, North Carolina and currently live in Durham, North Carolina.

In 2019, as further described below, I along with Greg Herschlag and other collaborators developed a mathematical algorithm to implement the county clustering rules in *Stephenson v. Bartlett*. We made that algorithm and the source code freely available online. In 2021, we published a paper that applied the algorithm to determine the possible arrangements of county clusters that satisfied the *Stephenson* clustering rules using the results of the 2020 Census. (We did not take account of the VRA and in effect assumed for purposes of that paper, without analyzing the issue or making any conclusion, that no district was required to be drawn under the Voting Rights Act). I understand that the legislature relied on the clustering options in our paper when it drew North Carolina's House and Senate districts.

In 2023, we posted a version of the algorithm online that allowed users to first draw a district (such as a district required by the Voting Rights Act), freeze that district, and then implement the *Stephenson* clustering procedure. These abilities were always present in the algorithm, but the revised version of the algorithm made them easier to implement.

My curriculum vitae is provided in the appendix. I am being compensated at the rate of $400 per hour for work in this case. My compensation does not depend on the opinions I render. My prior expert engagements are set forth in my CV.

**<u>Background on *Stephenson* and our Public Algorithm and Code Applying the *Stephenson* Procedure</u>**

In *Stephenson v. Bartlett*, the North Carolina Supreme Court interpreted provisions in North Carolina's constitution relating to preserving counties in the drawing of state legislative districts. The Constitution's Whole County Provisions state that counties shall not be divided in the creation of a legislative district, but sometimes it is necessary to split counties to comply with the "one person, one vote" mandate of the U.S. Supreme Court. In *Stephenson* and *Dickson v. Rucho*, the North Carolina Supreme Court addressed this

issue and laid out a procedure for implementing the Whole County Provisions by combining North Carolina counties into county clusters, each of which contains one or more districts that are wholly contained within the cluster's boundaries.

*Stephenson* stated that "legislative districts required by the VRA shall be formed prior to creation of non-VRA districts." It then stated that "in counties having a 2000 census population sufficient to support the formation of one non-VRA legislative district falling at or within plus or minus five percent deviation from the ideal population consistent with 'one-person, one-vote' requirements, the WCP requires that the physical boundaries of any such non-VRA legislative district not cross or traverse the exterior geographic line of any such county."

Next, "[w]hen two or more non-VRA legislative districts may be created within a single county, which districts fall at or within plus or minus five percent deviation from the ideal population consistent with 'one-person, one-vote' requirements, single-member non-VRA districts shall be formed within said county."

Next, "[i]n counties having a non-VRA population pool which cannot support at least one legislative district at or within plus or minus five percent of the ideal population for a legislative district or, alternatively, counties having a non-VRA population pool which, if divided into districts, would not comply with the at or within plus or minus five percent "one-person, one-vote" standard, the requirements of the WCP are met by combining or grouping the minimum number of whole, contiguous counties necessary to comply with the at or within plus or minus five percent "one-person, one-vote" standard."

Districts are then formed within those multi-county groupings.

In *Dickson* in 2015, the North Carolina Supreme Court provided further guidance for implementing the Whole County Provisions. *Dickson* stated that

> "[T]he process established by this Court in *Stephenson I* and its progeny requires that, in establishing legislative districts, the General Assembly first must create all necessary VRA districts, single-county districts, and single counties containing multiple districts. Thereafter, the General Assembly should make every effort to ensure that the maximum number of groupings containing two whole, contiguous counties are established before resorting to groupings containing three whole, contiguous counties, and so on."

The *Stephenson* ruling translates into a greedy algorithm[1] that works to maximize the number of county clusters (i.e., groupings), which implies a low number of county

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[1] The term greedy refers to a strategy in optimization algorithms which makes the best "local" choice (e.g. maximizing the number of two-county clusters) in search of optimizing a more global target (e.g. maximizing the total number of clusters). The principal of local optimization (or greed) is used in a large collection of standard algorithms.

splits. The algorithm does not necessarily produce one unique combination of county clusters. Often it may be the case that different combinations of county clusters will comply equally well with the *Stephenson* procedure.

In 2019, we produced a free, publicly available, open-source algorithm to determine all optimal county clusterings that complied with the Court's guidance in Stephenson and Dickson. We described the algorithm and the mathematical theorems supporting it in a paper published in the journal Statistics and Public Policy in 2020.[2] We also provided a publicly accessible repository for our source code. This allowed anyone to run our algorithm.

In short, the algorithm converted the *Stephenson* rules about clustering into mathematical rules implemented by computer code. The algorithm first identified which single counties could be divided into an integer number of districts (i.e., one, two, three, etc.), then looked for all pairs of contiguous counties that contain an integer number of districts within 5% of the ideal population, where remaining counties can still be grouped into legal clusters. This process is then performed with groups of three contiguous counties, then four, and so on. Eventually, all of the counties are placed in a county cluster, with each cluster having an assigned number of districts it should be subdivided into. At each step, before accepting a particular cluster, the algorithm uses a search tree to confirm that it is still possible to cluster all remaining unassigned counties into legal clusters.

After the 2020 Census, we applied this algorithm to the North Carolina population data in anticipation of the approaching redistricting cycle,[3] and produced a paper that provided clustering options for North Carolina based on the 2020 census results. That paper is attached as Appendix 1. We noted in that paper that our application of the Stephenson procedure did not take account of the VRA and in effect assumed that no VRA district was required. Whether a VRA district is required by the law is beyond our competence.

It is our understanding that (i) the North Carolina General Assembly determined that no VRA districts were needed, and (ii) the General Assembly then relied on the clustering options described in our paper to determine the possible county clusters available under *Stephenson*.

Notably, while our algorithm is guaranteed to maximize the number of county clusters within the *Stephenson* framework, it is not guaranteed to minimize the number of split counties overall. The procedure in *Stephenson* and *Dickson* prioritizes creating clusters with the fewest number of counties, not minimizing county splits overall, and

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[2] Optimal Legislative County Clustering in North Carolina. Daniel Carter, Zach Hunter, Dan Teague, Gregory Herschlag, and Jonathan Mattingly. Statistics and Public Policy, Volume 7, 2020.
[3] https://sites.duke.edu/quantifyinggerrymandering/files/2021/08/countyClusters2020.pdf

creating clusters with the fewest number of counties does not always result in minimizing county splits. Thus, the algorithm is not guaranteed to minimize the number of counties that are split. Put differently, in the absence of the *Stephenson* procedure, it would often be possible to draw a districting map that splits fewer counties than are split under a map that complies with *Stephenson*.

Work by researchers at Oklahoma State provides a demonstration of this. Those researchers developed a different algorithm, which they applied to every state, to draw state legislative maps that comply with population and contiguity requirements while minimizing the number of county splits.[4] When they applied that algorithm after the 2020 Census to generate county groupings for the North Carolina Senate, they produced certain groupings[5] and a map using those groupings[6] that required splitting of only 13 counties, rather than the 15 counties that are split in the enacted 2023 North Carolina Senate map. The groupings the Oklahoma State researchers generated are not compliant with *Stephenson*, however, because it is possible to create more two-county groupings than are contained in their map, and the North Carolina Supreme Court has instructed the legislature to ensure that the "maximum number of groupings containing two whole, contiguous counties are established before resorting to groupings containing three whole, contiguous counties, and so on." The clustering that their code generated contained 7 two-county groupings, while the clustering options that our algorithm generated and that the General Assembly used contained 9 two-county groupings.

## Accounting for Districts That May Be Required by the VRA

In fall of 2023, as part of our research, we modified the code in our public repository to make it easier to run the algorithm while freezing particular counties. This possibility always existed in the algorithm; however, we modified the code to make this ability more explicit.

The modified code allows the user to explicitly freeze several counties. These frozen counties could be used to either form a VRA compliant district or a cluster in which a VRA district has been formed. The modified code assumes that the frozen counties can be grouped into one or more districts within the 5% population tolerance. Frozen counties are removed from consideration and the standard Stephenson algorithm is run on the remaining counties. We published that modified code online on October 3, 2023.

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[4] Maral Shahmizad and Austin Buchanan, Political Districting to Minimize County Splits, https://www.researchgate.net/profile/Austin-Buchanan/publication/368662604_Political_districting_to_minimize_county_splits/links/63f3c9af0cf1030a563a068f/Political-districting-to-minimize-county-splits.pdf

[5] https://github.com/maralshahmizad/Political-Districting-to-Minimize-County-Splits/blob/main/cluster_png/NC_SS_clusters.png

[6] https://davesredistricting.org/maps#viewmap::715dea51-c5f8-4407-b927-5fdb8e165b21

Subsequently, the Plaintiffs' counsel in this case asked us to further modify the code to allow for the ability to freeze a district that splits a county, and then to apply the *Stephenson* procedure to the remaining portion of the map to obtain compliant county groupings. The unused section of the split county is left in the pool to be joined with other counties through the Stephenson procedure.

When only part of a county is frozen, there are two possible ways of implementing the Stephenson guidelines.

One option is to treat the non-frozen portion of the county as if it were itself a whole county, such that the algorithm would treat a cluster that joined the non-frozen portion of a county to an adjoining whole county as a two-county cluster, or that joined the non-frozen portion of the county to two adjoining whole counties as a three-county cluster, etc. Under that approach, for example, if the non-frozen portion of the county could be combined with another whole county to form a district within the population constraints, the algorithm would treat that as a two-county cluster and select it before going on to consider three-county clusters throughout the rest of the state.

A second option is to treat the non-frozen portion of the county as if it were part of a cluster containing the entirety of the frozen district, rather than as its own county. Under that circumstance, if the non-frozen portion of the county could be combined with one other whole county to form a compliant district, that combination would not be treated as its own two-county cluster and would not be preferred to clusters containing three full counties. Rather, the resulting cluster would be treated as if it contained all the counties in the frozen district.

We developed code to implement both versions. As described below, Plaintiffs' counsel asked us to run both versions of the algorithm three times, each time freezing a different district that splits one county.[7] In all three cases, we obtained the same clustering options regardless of which version of the algorithm we used.

## Application of *Stephenson* Algorithm to Plaintiffs' Demonstration Districts

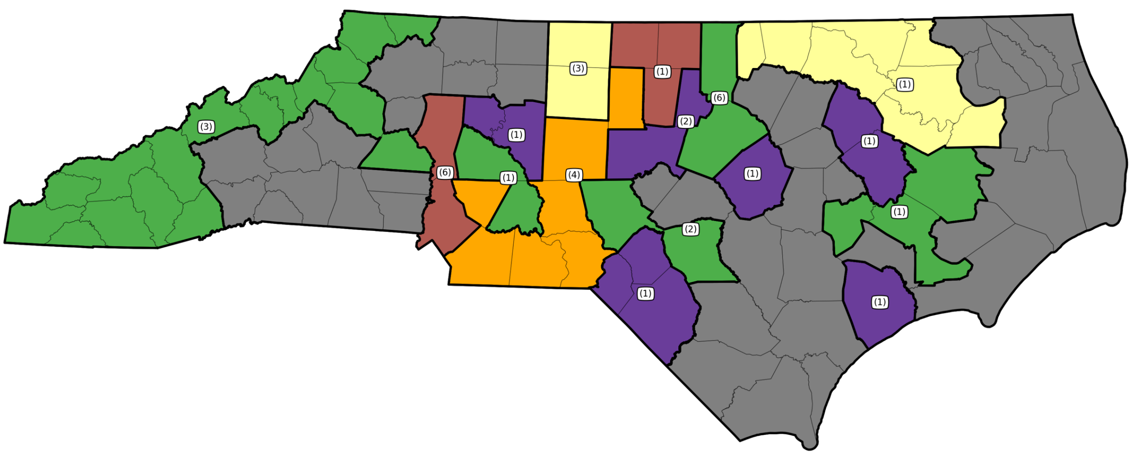
We were asked in this litigation to apply our algorithm in a way that accounts for the VRA. Counsel for the plaintiffs gave us four districts, each one centered in northeastern North Carolina, told us to assume that each was a VRA district, and asked us to apply our algorithm to produce the clustering options that complied with *Stephenson*.

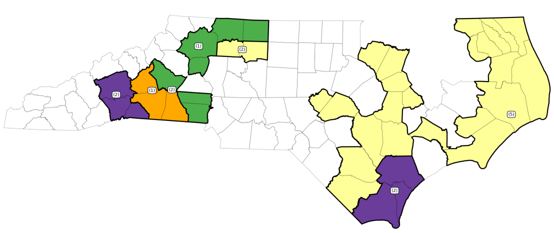
[7] Although there were four total demonstration districts, only three contained a split county.

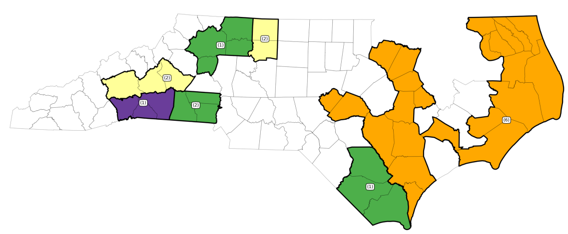
**Demonstration District A**

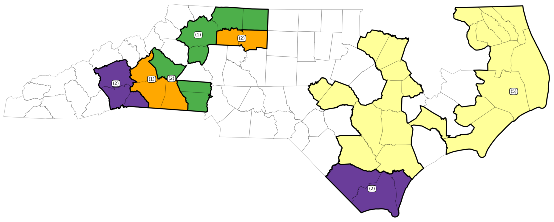
This district, which we understand Plaintiffs' counsel is referring to as Demonstration District A, is comprised of the counties Vance, Warren, Halifax, Northampton, Hertford, Bertie, Martin, and Washington, each in their entirety. Plaintiffs also told us to freeze the Pitt-Edgecombe single district cluster. We then applied the *Stephenson* algorithm to the remaining counties. We find that there are 12 possible county clusters. There are three distinct regions of clusters that have independent choices of clusters: The two Western regions have two possible choices each, whereas the Eastern region has three possible choices. The two, two, and three independent choices lead to 12 overall possible choices.

We first display the county clusters that are consistent across all 12 choices. We then display the independent choices. The two figures on the left show the two choices in the two Western regions and two of the choices in the Eastern region. The figure on the right shows the third county cluster choice in the Eastern region while showing redundant choices in the two Western regions.









We remark that the Western county clusters may be chosen to be identical to those used in the clusters in the enacted 2023 map. In other words, the *Stephenson*-compliant options for the Western county clusters are the same as the *Stephenson*-compliant options that resulted when we applied the algorithm in our 2020 paper and effectively assumed that no VRA district was required.

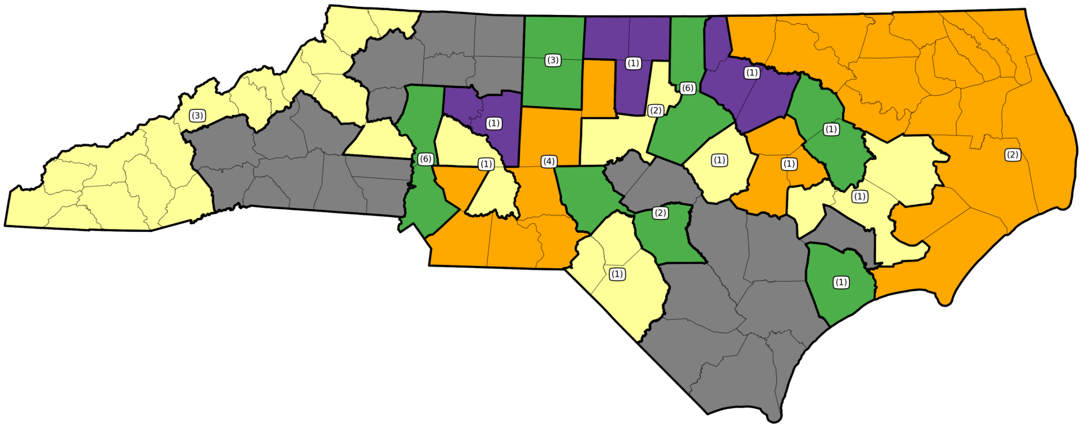
In all 12 of the county cluster options, at least 17 counties must be split when drawing districts.

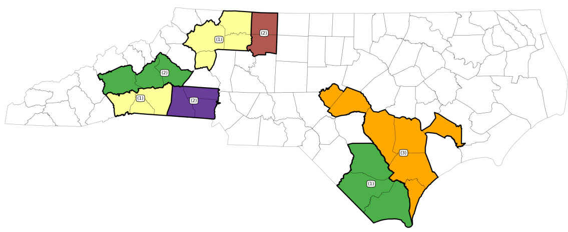
**DEMONSTRATION DISTRICT B**

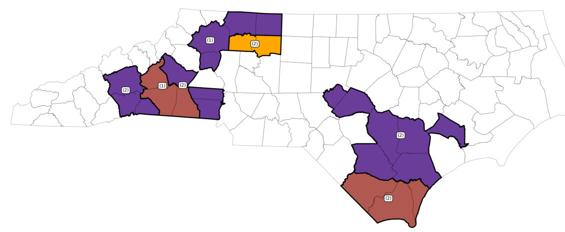
As a second investigation, we were also asked to consider a frozen district consisting of Chowan, Warren, Gates, Halifax, Bertie, Northampton, Martin, Hertford, and part of Pasquotank county. The district included six of nine precincts in Pasquotank: Newland, East, North, Providence, South, and West. It did not include Mt Hermon, Nixonton, and Salem.

As mentioned above, there are two ways to approach clustering the remaining part of the state when a frozen district splits a county. We perform both versions of the algorithm and find that the resulting county cluster options are identical. We find that there are 8 possible county clusters, because there are three regions in the state that each have two independent choices for clusters. In all cases, 16 counties must be split when drawing districts.

Again, we begin by displaying the county clusters that are consistent across all 8 county cluster options. We then present two figures each that contain one of two unique and independent choices for the three regions of the state.







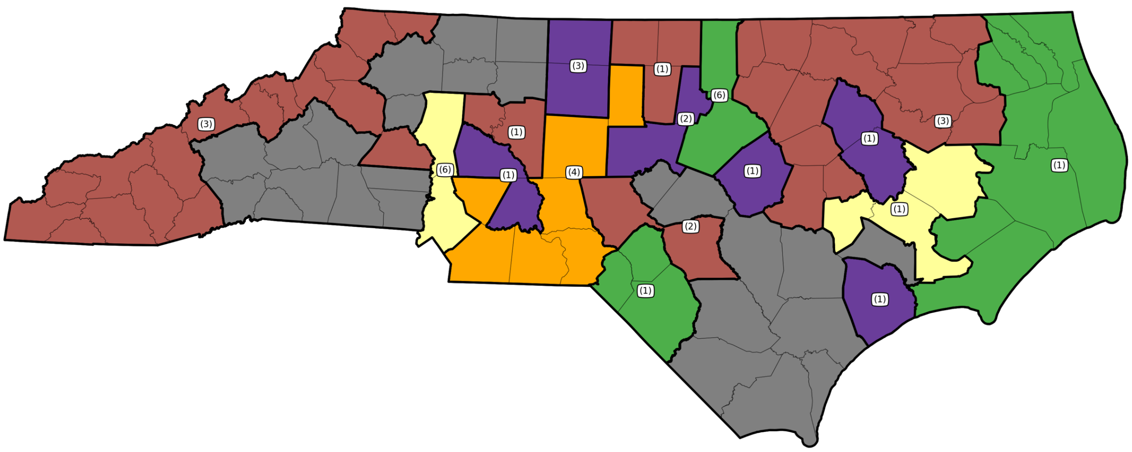
We remark that, with the exception of the two-district cluster in the Eastern portion of the map, all other county clusters may be chosen to be identical to those used in the clusters in the enacted 2023 map. In other words, the *Stephenson*-compliant options for 48 of the districts are the same as the *Stephenson*-compliant options that resulted when we applied the algorithm in our 2020 paper and effectively assumed that no VRA district was required.

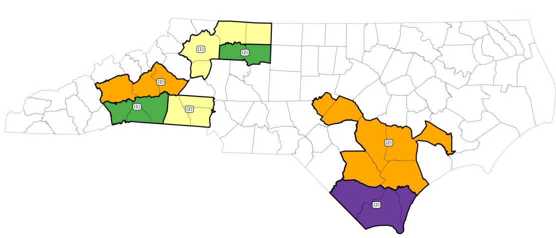
**Demonstration District C**

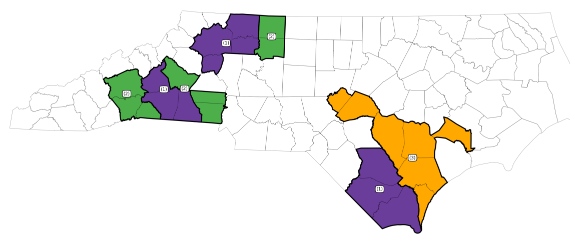
As a third investigation, we were asked to consider a frozen district consisting of Hertford, Northampton, Chowan, Halifax, Martin, Gates, Washington, Bertie, Warren, and part of Vance county. The district included 8 of 12 precincts in Vance County: East Henderson 1, Hill Top, Middleburg, North Henderson 1, Northern Vance, South Henderson 1, South Henderson 2, and West Henderson. It did not include Community College, Kittrell, New Hope, and Sandy Creek.

We again approach clustering the remaining part of the state in two ways when a frozen district splits a county. We perform both versions of the algorithm and find that the resulting county cluster options are identical. We find that there are 8 possible county cluster options, because there are three regions in the state that each have two independent choices for clusters. In all cases, 17 counties must be split when drawing districts.

We again begin by displaying the county clusters that are consistent across all 8 county cluster options. We then present two figures each that contain one of two unique and independent choices for the three regions of the state.







We remark that the Western-most and Southern county clusters may be chosen to be identical to those used in the clusters in the enacted 2023 map. In other words, the *Stephenson*-compliant options for the Western-most and Southern county clusters are the same as the *Stephenson*-compliant options that resulted when we applied the algorithm in our 2020 paper and assumed that no VRA district was required.

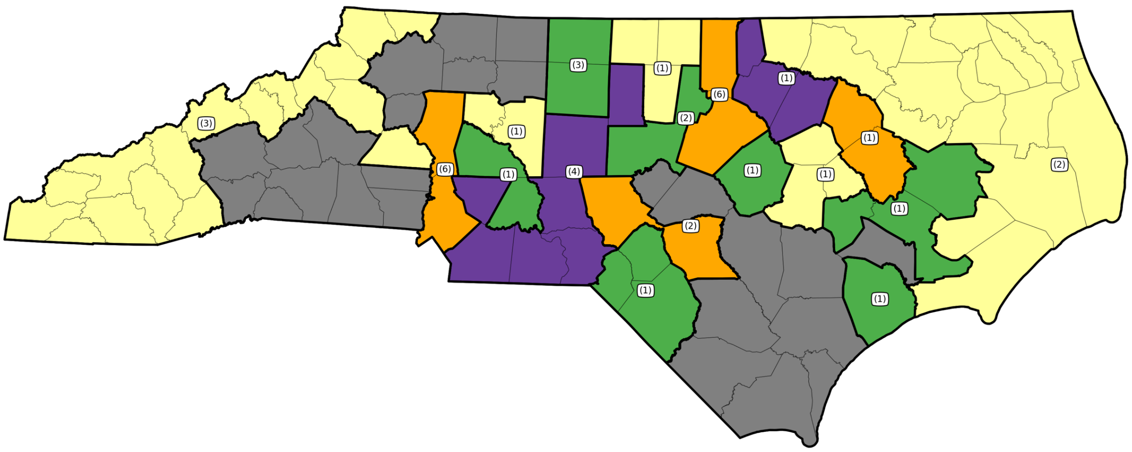
**Demonstration District D**

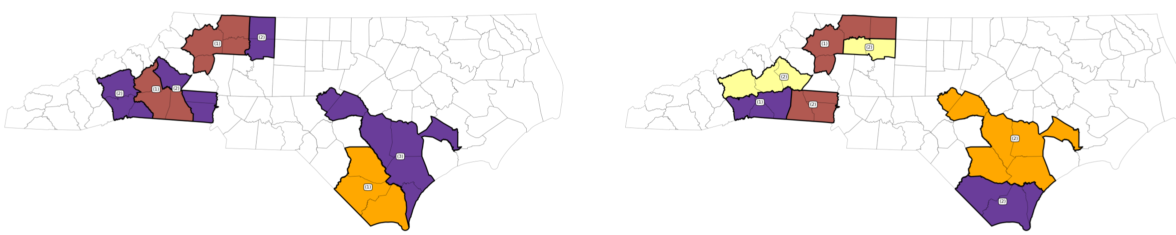
As a fourth investigation, we were asked to consider a frozen district consisting of Bertie, Gates, Halifax, Hertford, Martin, Northampton, Tyrrell, Warren, Washington, and part of Pasquotank county. The district included six of nine precincts in Pasquotank: Newland, East, North, Providence, South, and West. It did not include Mt Hermon, Nixonton, and Salem.

We again approach clustering the remaining part of the state in two ways when a frozen district splits a county. We perform both versions of the algorithm and find that the resulting county cluster options are identical. We find that there are 8 possible county

cluster options, because there are three regions in the state that each have two independent choices for clusters. In all cases, 16 counties must be split when drawing districts.

We again begin by displaying the county clusters that are consistent across all 8 county cluster options. We then present two figures each that contain one of two unique and independent choices for the three regions of the state.





We remark that, with the exception of the two-district cluster in the Eastern portion of the map, all other county clusters may be chosen to be identical to those used in the clusters in the enacted 2023 map. In other words, the *Stephenson*-compliant options for 48 of the districts are the same as the *Stephenson*-compliant options that resulted when we applied the algorithm in our 2020 paper and effectively assumed that no VRA district was required.

I declare under penalty of perjury that the foregoing is true and correct.
Executed on May 31, 2024

Jonathan Mattingly

Appendix 1

# NC General Assembly County Clusterings from the 2020 Census

Christopher Cooper[1], Blake Esselstyn[2], Gregory Herschlag[3],
Jonathan Mattingly[3], Rebecca Tippett[4]

In the North Carolina General Assembly districting process, county clusters are used to minimize the overall number of county splits while maintaining population balance in the redistricting process. Determining the county clusters for the NC House and for the NC Senate is the first step in the redistricting process for the NC General Assembly. The county clusters are largely algorithmically determined through an optimization procedure outlined by the NC Supreme Court in [Stephenson v. Bartlett](). However there are often multiple optimal county clusterings that minimize county splitting (see [the Quantifying Gerrymandering blog]() and [the Districks.com explainer]() for more details). The release of the 2020 census data allows us to determine the possible county clusterings for both the North Carolina State House and State Senate redistricting processes. The one part of Stephenson v. Bartlett which this analysis does not reflect is compliance with the Voting Rights Act. To determine the county clusters, we used the implementation of the court order procedure described in Cater et al.[5]

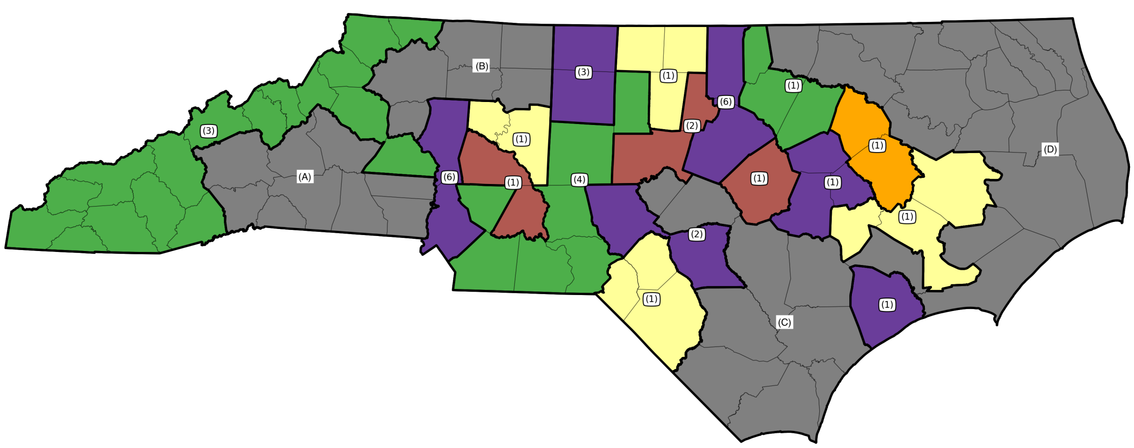



Figure 1: The NC Senate clusters that are fixed shown as colored regions annotated with a number in parentheses giving the number of districts the cluster contains. The four grayed-out regions (labeled A-D) each contain two alternative clusterings. The different options of the grayed-out regions are given in Figure 2. One may mix and match different choices from each of the two options which yields a total of 16 different county clustering maps.

[1] Political Science and Public Affairs, Western Carolina University
[2] FrontWater, LLC and Mapfigure Consulting
[3] Duke Mathematics Department and the Quantifying Gerrymandering Project, Duke University. We thank Alexis Sparko for help with map visualization.
[4] Carolina Demography, UNC at Chapel Hill
[5] *Optimal Legislative County Clustering in North Carolina.* Daniel Carter, Zach Hunter, Dan Teague, Gregory Herschlag, and Jonathan Mattingly. Statistics and Public Policy, Volume 7, 2020

## NC State Senate County Clusterings

In the state Senate, there are 17 clusters containing 36 of the 50 districts that are fixed based on determining optimal county clusters. These are represented by the colored county groupings in Figure 1. The white numbers annotating each county clustering give the number of districts that county cluster should contain. Ten of these clusters contain one district, meaning that ten of the 50 senate districts are fixed (i.e. these will be the official districts in the coming cycle). The remaining county clusters must be further subdivided into legislative districts in the coming redistricting process in the General Assembly.

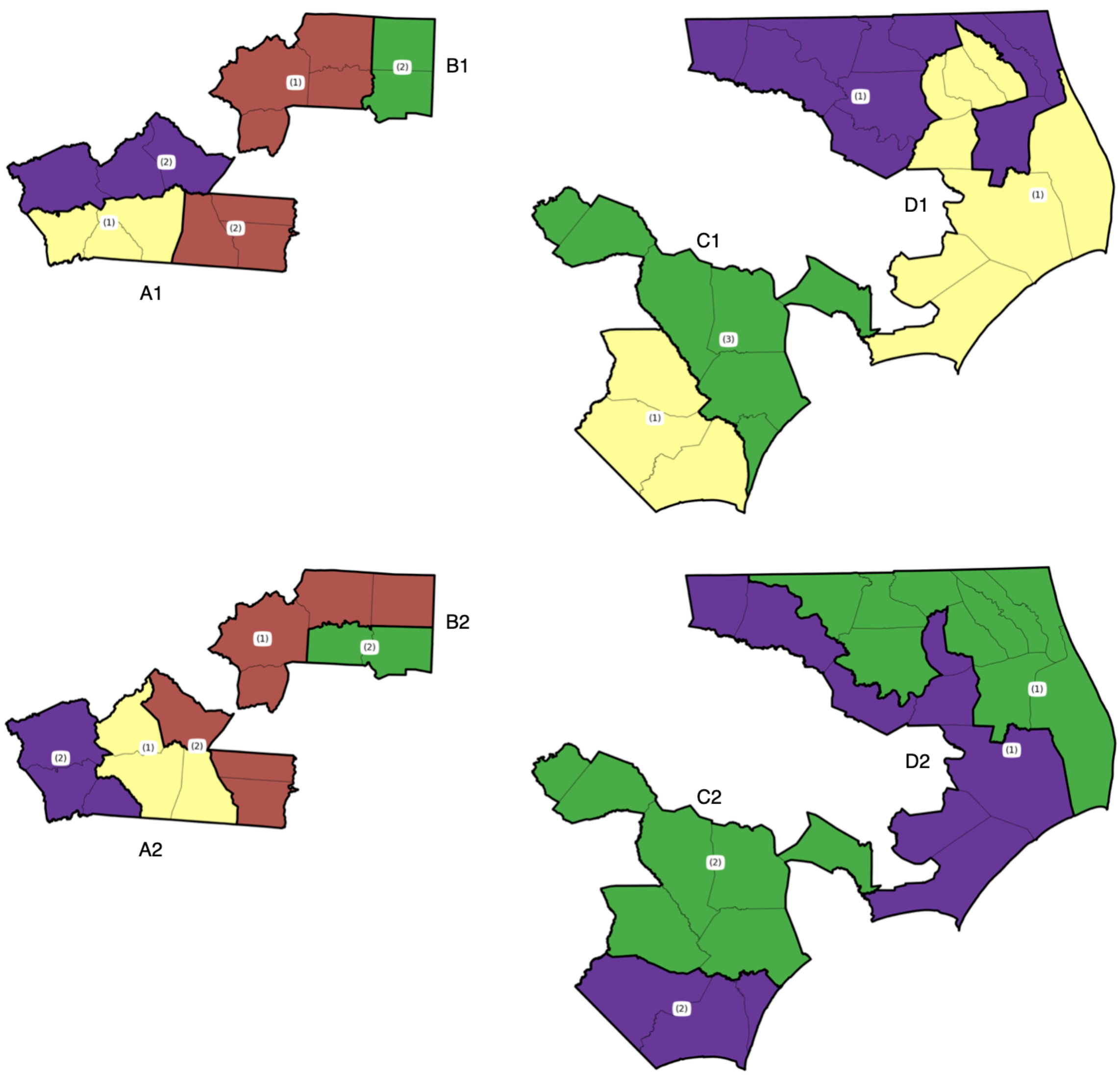



Figure 2: The two possible options in regions A, B, C and D of the NC Senate County Clusterings (top and bottom). The options from the two figures may be mixed. For example, a Senate clustering may be comprised of the fixed clusters from Figure 1, along with options A1, B2, C2, and D1. Again, the numbers in parentheses give the number of districts contained in each cluster.

The remaining clusters (shown in gray) are separated into four groups. Each group has two possible clusterings that minimize county splitting. In combination, there are 16 total possible statewide county clusterings. For simplicity of discussion, we have labeled the different regions where a choice exists as A, B, C, or D and denoted the two choices for each region as 1 or 2. Hence A1 and A2 are the two choices for the A region. No preference is intended by the 1 versus 2 labeling.

The two options in each of the four regions are shown in Figure 2.

In region A to the southwest, Buncombe County may be paired either with McDowell and Burke Counties (A1), or with Henderson and Polk Counties (A2). In both cases, the cluster would be comprised of two districts, however, A2 necessitates that Burke County is paired with Gaston and Lincoln Counties through a very narrow connection which may impede compactness considerations. Furthermore, the Lincoln-Cleveland-Gaston cluster in A1 also exists in the current map. This may mean that the A1 southwestern cluster may be perceived as the more favorable option over A2 since it (i) provides an opportunity to create more compact districts and (ii) may provide an opportunity to draw districts that are nearly identical to the ones that exist in the in Lincoln-Cleveland-Gaston cluster (conditioned on fluctuations in the population).

In region B to the northwest, Forsyth County may either be paired with Stokes (B1) or Yadkin (B2); the remaining county (either Yadkin or Stokes) would then be paired with Surry, Wilkes, and Alexander Counties. In region C to the south, Brunswick and Columbus may be paired either with Bladen to create a one-district cluster (C1) or with New Hanover to create a two-district cluster (C2). Finally, in region D to the east, Carteret, Pamlico, Washington, Chowan, and Hyde Counties may either be paired with Dare, Perquimans and Pasquotank Counties (D1), or with Martin, Halifax and Warren Counties (D2).

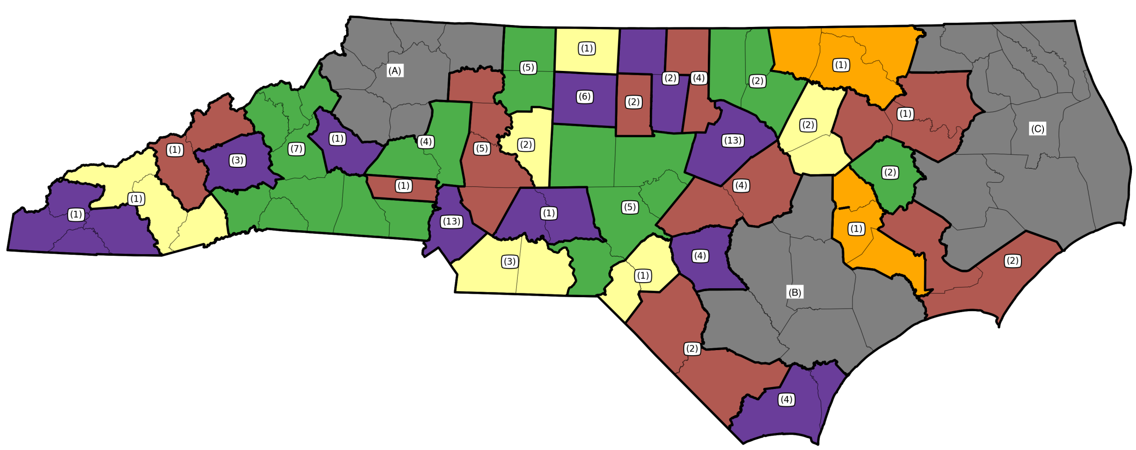



Figure 3: The NC House clusters that are fixed; there are three grayed-out regions (labeled A-C) that each contain two alternative clusterings. The different options of the grayed-out regions are given in Figure 4. One is free to mix and match different choices from the two options which yields a total of eight different county clustering maps.

## NC State House County Clusterings

In the state House, there are 33 clusters containing 107 of the 120 districts that are fixed based on determining optimal county clusters. These are represented by the colored county groupings in Figure 2. Again, the white numbers annotating each county clustering give the number of districts that county cluster should contain. Eleven of these clusters contain one district, meaning that eleven of the 120 house districts are fixed (i.e., these will be the official districts in the coming cycle).

The remaining clusters (shown in gray) are separated into three groups. Each group has two possible clusterings that minimize county splitting. In combination, there are eight total possible statewide county clusterings in the house. The two options in each of the three regions are shown in Figure 4.

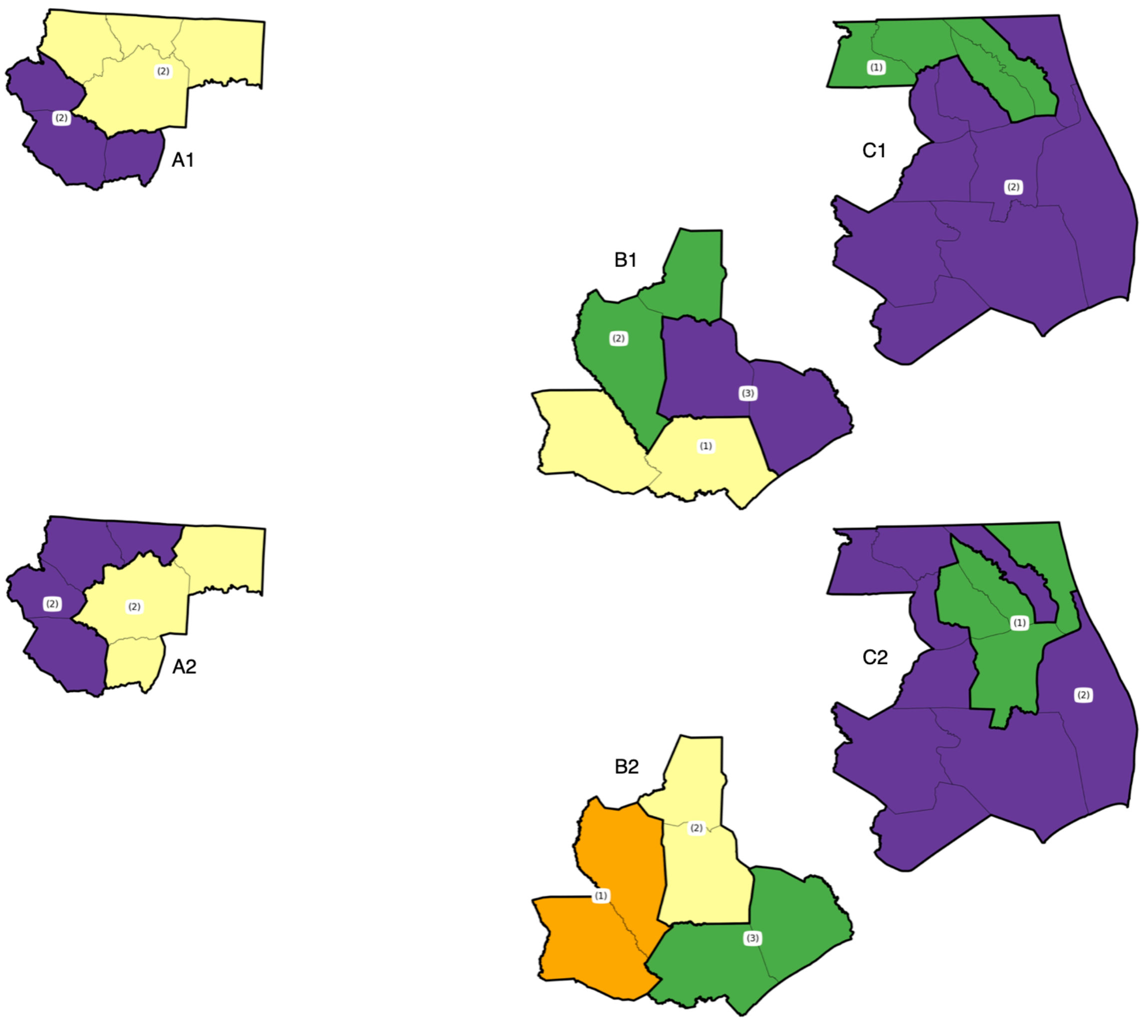



Figure 4: The two possible options in regions A, B, and C of the NC House County Clusterings (top and bottom). The options from the two figures may be mixed. For example, a House clustering may be comprised of the fixed clusters from Figure 3, along with options A2, B1, C2.

In region A to the northwest, Watauga and Caldwell may either be paired with Alexander (A1; purple) or with Ashe and Alleghany (A2; purple).

In region B to the south, Onslow may either be paired with Duplin (B1; purple) or with Pender (B2; green). The Duplin-Onslow cluster currently forms a three-district cluster and thus there may be an opportunity to minimally alter the three existing districts in this cluster (perhaps needing to adjust district boundaries based on population fluctuations). Because of this, B2 may end up as the selected clustering.

Finally, in region C to the east, either Currituck, Tyrell, Perquimans and Pasquotank will form a single district (C1), or Hertford, Gates, Camden and Pasquotank will form a single district (C2). In both cases, the remaining counties will form a cluster of two districts.

## Population Deviations

All the county clusterings are required to have populations such that the resulting districts are within 5% of the ideal district population, hence all the possible county clusters we have listed have population deviations less than 5%. In the Senate clusters, all possible choices of clusterings contain at least one district with a population deviation of more than 4.9%. In the House clusters, all possible choices of clusterings contain at least one district with a population deviation of 4.71%. Averaged across all the districts, all of the county clusterings have a mean deviation between 3.1% and 3.5% in the NC Senate and 1.2% and 1.5% in the NC House.

Tables 1 through Table 4 list each of the different county clusters contained in the different county clusterings. For each cluster, the relative average population deviation per district is given. Negative values indicate that the average district may be less populated than the ideal population size while positive values indicate that the average district will be more populated than the ideal population size.

The ideal population size is calculated by first taking the population of each cluster and dividing it by the number of districts in the cluster to obtain the average population per district for the cluster. The ideal district population is obtained by dividing the state population by the total number of districts (120 districts in the House and 50 districts in the Senate). The ideal population is then subtracted from the average population of a district in a cluster to obtain the deviation of the average cluster population from the ideal cluster population. This is then converted to a relative population deviation by dividing by the ideal population. It is this relative error, expressed as a percentage, which is reported in the table.

Tables 1 and 2 give the data for the different options for the NC Senate and NC House respectively. The clusters are grouped by the region label (A, B, C or D in the Senate and A, B, or C in the House). The labeling corresponds to that in the Figures in the preceding sections. Tables 3 and 4 give the data for the clusterings which are fixed in the Senate and House, respectively.

| NC Senate Clusters Which Vary Across Clusterings | Number of Districts | Option | 2020 Census Population | Average Population Deviation |
|---|---|---|---|---|
| Buncombe-Burke-McDowell | 2 | A1 | 401,600 | -3.83% |
| Cleveland-Gaston-Lincoln | 2 | A1 | 414,272 | -0.79% |
| Henderson-Polk-Rutherford | 1 | A1 | 200,053 | -4.18% |
| Buncombe-Henderson-Polk | 2 | A2 | 405,061 | -3.00% |
| Cleveland-McDowell-Rutherford | 1 | A2 | 208,541 | -0.12% |
| Burke-Gaston-Lincoln | 2 | A2 | 402,323 | -3.65% |
| Forsyth-Stokes | 2 | B1 | 427,110 | 2.28% |
| Alexander-Surry-Wilkes-Yadkin | 1 | B1 | 210,986 | 1.05% |
| Forsyth-Yadkin | 2 | B2 | 419,804 | 0.53% |
| Alexander-Stokes-Surry-Wilkes | 1 | B2 | 218,292 | 4.55% |
| Bladen-Brunswick-Columbus | 1 | C1 | 216,922 | 3.90% |
| Duplin-Harnett-Jones-Lee-New Hanover-Pender-Sampson | 3 | C1 | 599,681 | -4.26% |
| Bladen-Duplin-Harnett-Jones-Lee-Pender-Sampson | 2 | C2 | 403,585 | -3.35% |
| Brunswick-Columbus-New Hanover | 2 | C2 | 413,018 | -1.09% |
| Carteret-Chowan-Dare-Hyde-Pamlico-Pasquotank-Perquimans-Washington | 1 | D1 | 199,750 | -4.33% |
| Bertie-Camden-Currituck-Gates-Halifax-Hertford-Martin-Northampton-Tyrrell-Warren | 1 | D1 | 198,430 | -4.96% |
| Carteret-Chowan-Halifax-Hyde-Martin-Pamlico-Warren-Washington | 1 | D2 | 198,557 | -4.90% |
| Bertie-Camden-Currituck-Dare-Gates-Hertford-Northampton-Pasquotank-Perquimans-Tyrrell | 1 | D2 | 199,623 | -4.39% |

Table 1: This table gives the NC Senate Clusters which vary across the 16 different possible clusterings of the entire state. The different clusterings are formed by choosing either option 1 or 2 from the four different regions (A, B, C, and D).

| NC House Clusters Which Vary Across Clusterings | Number of Districts | Option | 2020 Census Population | Average Population Deviation |
|---|---|---|---|---|
| Alexander-Surry-Wilkes | 2 | A1 | 173,772 | -0.13% |
| Alleghany-Ashe-Caldwell-Watauga | 2 | A1 | 172,203 | -1.03% |
| Alexander-Caldwell-Watauga | 2 | A2 | 171,182 | -1.61% |
| Alleghany-Ashe-Surry-Wilkes | 2 | A2 | 174,793 | 0.46% |
| Bladen-Pender | 1 | B1 | 89,809 | 3.23% |
| Duplin-Onslow | 3 | B1 | 253,291 | -2.95% |
| Sampson-Wayne | 2 | B1 | 176,369 | 1.37% |
| Bladen-Sampson | 1 | B2 | 88,642 | 1.89% |
| Duplin-Wayne | 2 | B2 | 166,048 | -4.56% |
| Onslow-Pender | 3 | B2 | 264,779 | 1.45% |
| Beaufort-Chowan-Currituck-Dare-Hyde-Pamlico-Perquimans-Tyrrell-Washington | 2 | C1 | 167,493 | -3.73% |
| Camden-Gates-Hertford-Pasquotank | 1 | C1 | 82,953 | -4.65% |
| Beaufort-Camden-Chowan-Dare-Gates-Hertford-Hyde-Pamlico-Washington | 2 | C2 | 165,528 | -4.86% |
| Currituck-Pasquotank-Perquimans-Tyrrell | 1 | C2 | 84,918 | -2.39% |

Table 2: This table gives the NC House Clusters which vary across the eight different possible clusterings of the entire state. The different clusterings are formed by choosing option 1 or 2 from the 3 different regions (A, B, or C).

| NC Senate Clusters Which Are Fixed Across Clusterings | Number of Districts | 2020 Census Population | Average Population Deviation |
|---|---|---|---|
| Iredell-Mecklenburg | 6 | 1,302,175 | 3.95% |
| Granville-Wake | 6 | 1,190,402 | -4.98% |
| Alamance-Anson-Cabarrus-Montgomery-Randolph-Richmond-Union | 4 | 870,409 | 4.22% |
| Guilford-Rockingham | 3 | 632,395 | 0.96% |
| Alleghany-Ashe-Avery-Caldwell-Catawba-Cherokee-Clay-Graham-Haywood-Jackson-Macon-Madison-Mitchell-Swain-Transylvania-Watauga-Yancey | 3 | 642,393 | 2.56% |
| Chatham-Durham | 2 | 401,118 | -3.94% |
| Cumberland-Moore | 2 | 434,455 | 4.04% |
| Caswell-Orange-Person | 1 | 210,529 | 0.83% |
| Franklin-Nash-Vance | 1 | 206,121 | -1.28% |
| Johnston | 1 | 215,999 | 3.45% |
| Rowan-Stanly | 1 | 209,379 | 0.28% |
| Beaufort-Craven-Lenoir | 1 | 200,494 | -3.97% |
| Hoke-Robeson-Scotland | 1 | 202,786 | -2.87% |
| Edgecombe-Pitt | 1 | 219,143 | 4.96% |
| Davidson-Davie | 1 | 211,642 | 1.37% |
| Onslow | 1 | 204,576 | -2.02% |
| Greene-Wayne-Wilson | 1 | 216,568 | 3.73% |

Table 3: This table gives the NC Senate clusters which are fixed across all 16 of the possible clustering maps.

| NC House Cluster<br>Which Are Fixed Across Clusterings | Number of Districts | 2020 Census Population | Average Population Deviation |
|---|---|---|---|
| Mecklenburg | 13 | 1,115,482 | -1.37% |
| Wake | 13 | 1,129,410 | -0.13% |
| Avery-Cleveland-Gaston-Henderson-McDowell-Mitchell-Polk-Rutherford-Yancey | 7 | 623,272 | 2.35% |
| Guilford | 6 | 541,299 | 3.70% |
| Forsyth-Stokes | 5 | 427,110 | -1.81% |
| Chatham-Lee-Moore-Randolph-Richmond | 5 | 426,414 | -1.97% |
| Cabarrus-Davie-Rowan-Yadkin | 5 | 452,605 | 4.05% |
| Brunswick-New Hanover | 4 | 362,395 | 4.14% |
| Cumberland | 4 | 334,728 | -3.81% |
| Harnett-Johnston | 4 | 349,567 | 0.46% |
| Catawba-Iredell | 4 | 347,303 | -0.19% |
| Durham-Person | 4 | 363,930 | 4.58% |
| Anson-Union | 3 | 260,322 | -0.25% |
| Buncombe | 3 | 269,452 | 3.24% |
| Columbus-Robeson | 2 | 167,153 | -3.93% |
| Nash-Wilson | 2 | 173,754 | -0.14% |
| Carteret-Craven | 2 | 168,406 | -3.21% |
| Davidson | 2 | 168,930 | -2.91% |
| Franklin-Granville-Vance | 2 | 172,143 | -1.06% |
| Pitt | 2 | 170,243 | -2.15% |
| Alamance | 2 | 171,415 | -1.48% |
| Caswell-Orange | 2 | 171,432 | -1.47% |
| Rockingham | 1 | 91,096 | 4.71% |
| Bertie-Edgecombe-Martin | 1 | 88,865 | 2.15% |
| Lincoln | 1 | 86,810 | -0.21% |
| Hoke-Scotland | 1 | 86,256 | -0.85% |

| NC House Cluster<br>Which Are Fixed Across Clusterings | Number of Districts | 2020 Census Population | Average Population Deviation |
|---|---|---|---|
| Haywood-Madison | 1 | 83,282 | -4.27% |
| Greene-Jones-Lenoir | 1 | 84,745 | -2.59% |
| Jackson-Swain-Transylvania | 1 | 90,212 | 3.70% |
| Halifax-Northampton-Warren | 1 | 84,735 | -2.60% |
| Burke | 1 | 87,570 | 0.66% |
| Montgomery-Stanly | 1 | 88,255 | 1.45% |
| Cherokee-Clay-Graham-Macon | 1 | 84,907 | -2.40% |

Table 4: This table gives the NC House clusters which are fixed across all 8 of the possible clustering maps.

## Incumbents

We now perform a simple analysis of the effect of the new county clustering on the ability to preserve incumbencies. We do this, not to endorse or critique incumbency preservation, but because the NC General Assembly has identified it as one of its redistricting criteria. The new county clustering is only one way in which the new 2020 Census data influences the incumbency protection efforts. A more complete understanding of the effect on incumbency protection will require an analysis how geopolitical geography of the new Census data interacts with the redistricting process. We hope to investigate this more completely in the coming months.

For the moment, we simply note the number of incumbents in each county cluster (based on their official county of residence as obtained from the Redistricting Data Hub) and compare it to the number of districts each county clustering dictates. The following figures are repeats of the previous figures with an additional number added to the annotating white circles. The first number still gives the number of districts for each county cluster and the second number gives the number of incumbents currently residing in county cluster. When the first number is larger than the second, we outline the label in green to denote there is an opportunity to elect a new representative, assuming a current incumbent from another cluster does not relocate, even if all of the incumbents are re-elected.[6] When the second number is larger than the first, we outline the label in red to denote that at least one of the incumbents cannot be re-elected from this county cluster.

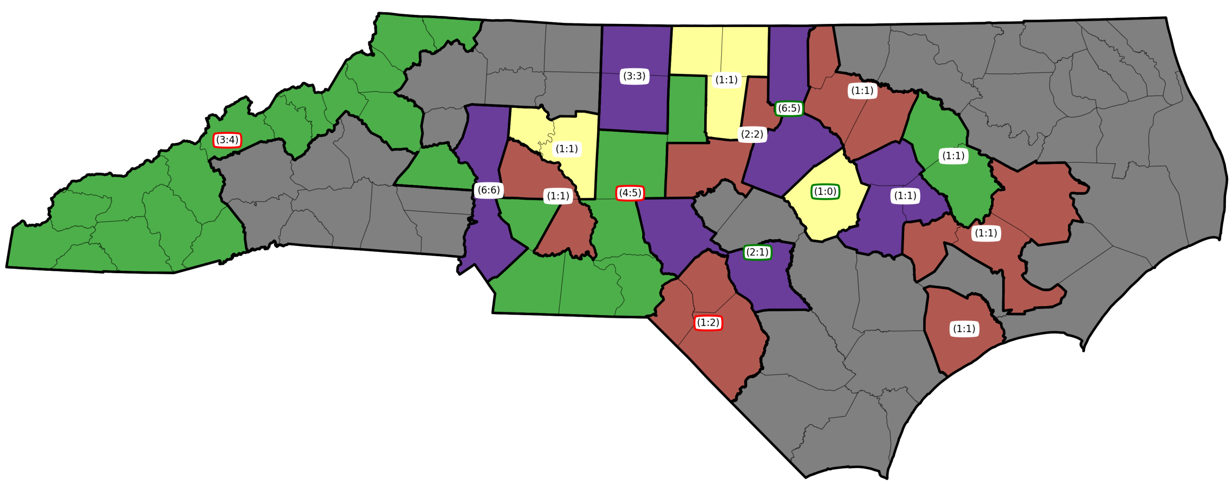



Figure 5: For the fixed clusters in the NC Senate, we display the number of districts followed by the number of incumbents within the cluster. Cluster labels highlighted in red must double bunk at least two incumbents. Cluster labels highlighted in green will elect at least one representative who is not currently serving in office.

[6] Candidates for the General Assembly must reside in their district at least once year prior to the general election.

Figure 5 highlights impacts in the NC Senate. The fixed clusterings in Johnston County, Wake-Granville, and Moore-Hoke will each elect at least one representative not currently serving in office. The following three fixed clusters will double bunk at least two incumbents:

- Alamance-Anson-Cabarrus-Montgomery-Randolph-Richmond-Union
- Alleghany-Ashe-Avery-Caldwell-Catawba-Cherokee-Clay-Graham-Haywood-Jackson-Macon-Madison-Mitchell-Swain-Transylvania-Watauga-Yancey
- Hoke-Robeson-Scotland

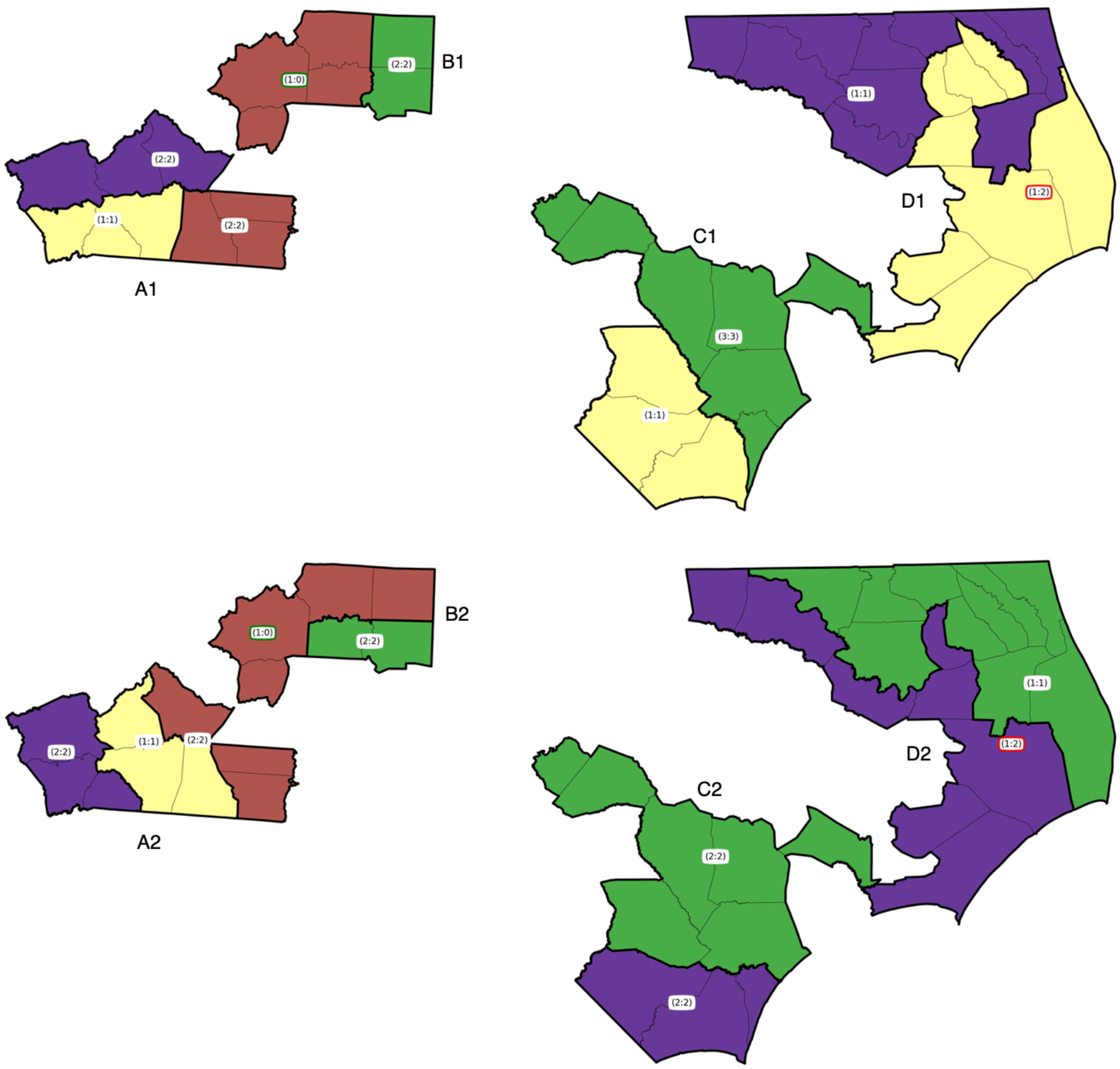



Figure 6: For the optional clusters in the NC Senate, we display the number of districts followed by the number of incumbents within the cluster. Cluster labels highlighted in red must double bunk at least two incumbents. Cluster labels highlighted in green will elect at least one representative who is not currently serving in office.

Figure 6 indicates that the clusters in region D produce a cluster that will double bunk two incumbents.

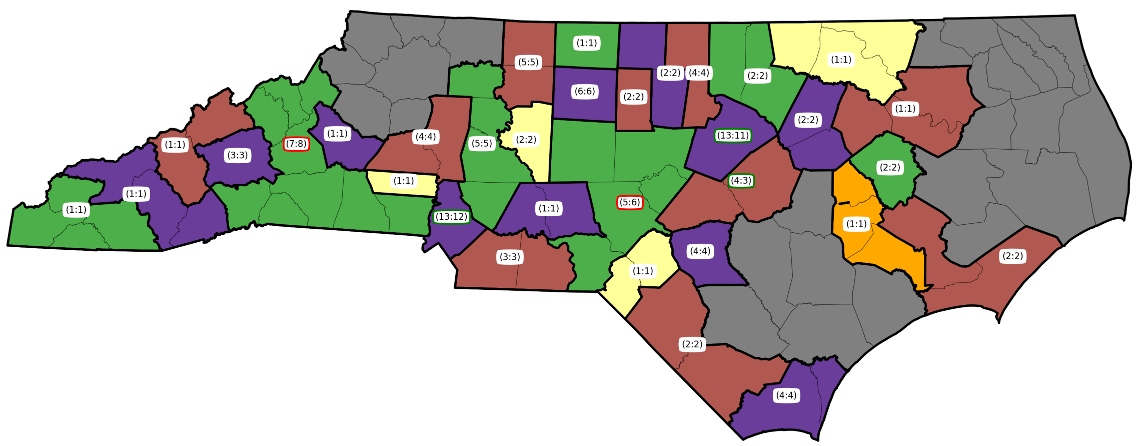


Figure 7: For the fixed clusters in the NC House, we display the number of districts followed by the number of incumbents within the cluster. Cluster labels highlighted in red must double bunk at least two incumbents. Cluster labels highlighted in green will elect at least one representative who is not currently serving in office.

Figure 7 highlights impacts of redistricting in the NC House. The fixed clusterings of Mecklenburg, Wake, and Harnett-Johnston will each elect at least one representative not currently serving in office. The following two fixed clusters will double bunk at least two incumbents:

- Avery-Cleveland-Gaston-Henderson-McDowell-Mitchell-Polk-Rutherford-Yancey
- Chatham-Lee-Moore-Randolph-Richmond

Figure 8 indicates that all options of potential clusters (A, B, and C) for the NC House will cause double bunking of at least two incumbents in two districts.

In addition to the above analysis, we also analyze the clusters with respect to minimizing county traversals. A county traversal occurs when a district extends over the boundary of two counties. Even though the number of incumbents may match the number of districts, it could still be impossible to draw districts that minimize county splitting and county traversals.

We have only discovered one cluster in which it is not possible to draw district boundaries while simultaneously minimizing traversals and preventing two incumbents being placed in the same newly formed district. This instance is in Cabarrus-Davie-Rowan-Yadkin House cluster in which Davie and Yadkin each hold an incumbent, however, the two counties do not have enough joint population to make up a single house district. Because of the geometry of the cluster, these two

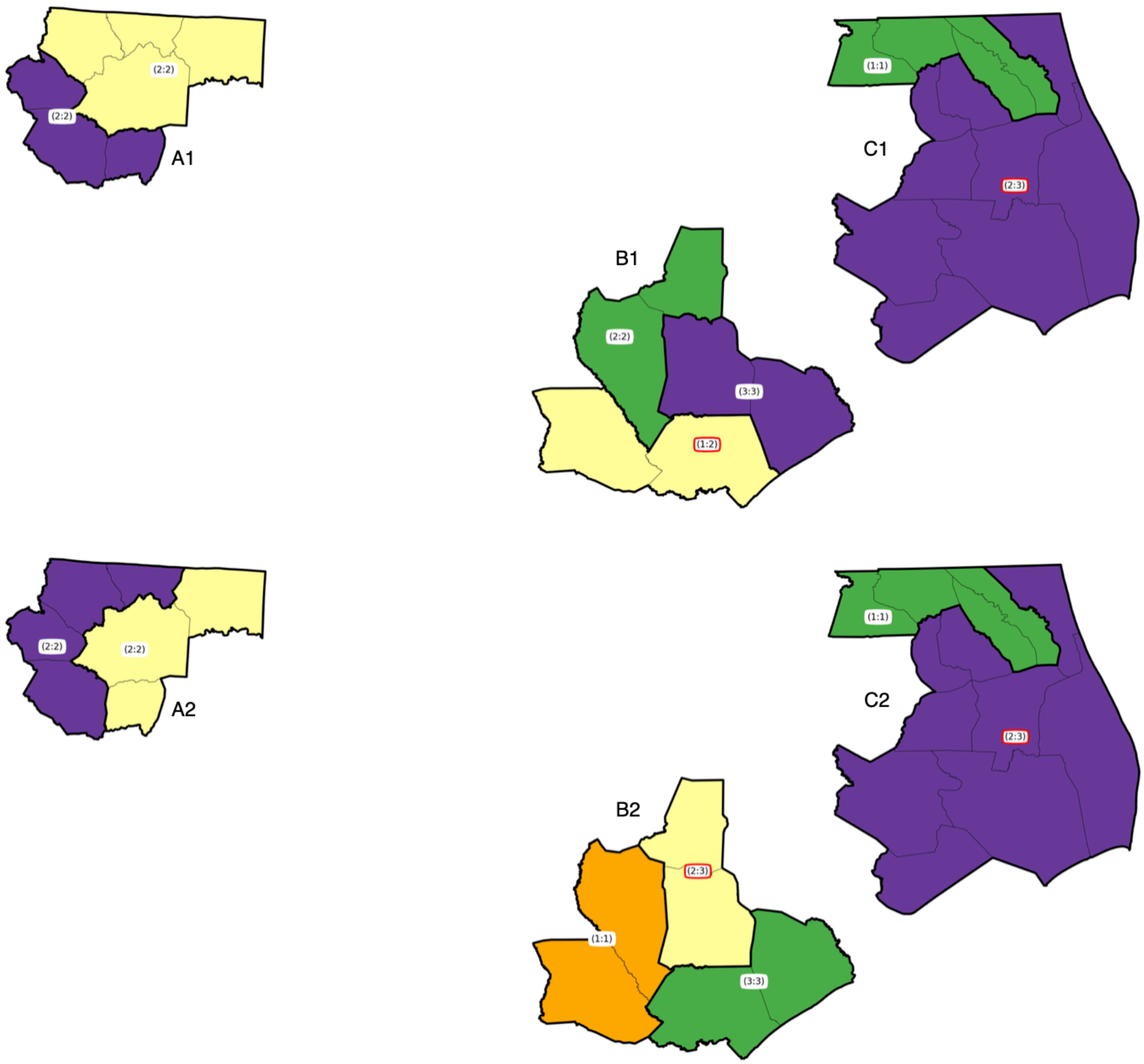



Figure 8: For the optional clusters in the NC House, we display the number of districts followed by the number of incumbents within the cluster. Cluster labels highlighted in red must double bunk at least two incumbents. Cluster labels highlighted in green will elect at least one representative who is not currently serving in office.

counties must then be combined as part of a single district ensuring the one of the two incumbents is not re-elected (see Figure 8 and the northern two counties within the 4-county 5:5 green cluster in the center of the state).

In aggregate, the NC Senate will contain four double bunked districts (regardless of the clustering options used), and the NC House will contain five double bunked districts (regardless of the clustering options used).

## Conclusion

Based on the 2020 Census, we have provided all of the possible county clusterings for the NC House and Senate obtain by the procedure outlined in *Stephenson v. Bartlett*. The consultants

associated with The Differentiators have announced that they have obtained the same groupings we have found using the software we released.

Although many of the clusters are now fixed, the General Assembly will be left to choose between various clustering options in some parts of the state. Certainly, compliance with the Voting Rights Act will be a key consideration in choosing between potential clusters. Preservation of communities of interest might also drive the decision to select one option over another. One could also consider choosing clusters to reduce the population deviations. For example, the B2 options in both the House and Senate clusterings have one district with a relative population deviation above 4.5%. As this necessitates that at least one of the districts in this cluster has a similarly large population deviation, it provides a reasonable rationale (if all other consideration are equal) to select the other clustering. There are clusterings with equally large deviations which might suggest choosing the alternative clustering option. One might also consider compactness, thought a less compact clustering, does not necessitate that the resulting districts are not compact. Hence this would need to be considered in each case.

We intend to follow this initial analysis with more in-depth looks at the clusterings and their implications.

# CURRICULUM VITAE

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Kimberly J. Jenkins Distinguished University Professor of New Technologies, Mathematics
Professor of Mathematics, Mathematics
Professor of Statistical Science, Statistical Science

**Education, Training and Certifications**

Ph.D., Princeton University 1998
M.A., Princeton University 1996
B.S., Yale University 1992

**Duke Appointment History**

James B. Duke Distinguished Professor, Mathematics 2019 - 2022
Chair of the Department of Mathematics, Mathematics 2016 - 2020
Associate Professor, Statistical Science 2008 - 2012
Associate Professor of Mathematics, Mathematics 2006 - 2012
Assistant Professor of Mathematics, Mathematics 2002 – 2005

**Other Academic Positions**

Visiting Professor, University of Pisa, Italy. 2023 - 2023
Simons Professor, SLMath, Berkeley CA. 2023 - 2023
Member, Institute for Advanced Study, Princeton NJ. 2021
Simons Professor, MSRI, Berkeley CA. 2015 – 2015
Visiting Professor, University de Nice. 2012 - 2012
Visiting Professor, University de Marseilles. 2010 - 2010
Visiting Professor, Berlin Summer School, TU Berlin. 2009 - 2009
Visiting Professor, University de Paris VI. 2008 - 2008
Principle Lecturer, Saint-Flour Summer School in Probability. 2007
Visiting Professor, MSRI, UC Berkeley. 2007 - 2007
Instructor Franco-Iranian Summer School, Zanjan, Iran. 2005
Visiting Member, Centro De Giorgi, SNS Pisa, Italy. 2006
Visiting Professor, University de Marseilles. 2002
Visiting Member, Institut Universitaire de France. 2003

Member special year in SPDE/Turbulence, Institute for Advance Study, Princeton. 2002 - 2003
NSF Post-Doctoral Fellow, Stanford University. 1999 – 2002
Szego Assistant Professor of Mathematics, Stanford University. 1998 – 2002
Visiting Scholar, Mathematics Institute, Warwick University. 2000 - 2000
Contractor, AT&T Shannon Labs. 1999 – 1999
Summer Intern, Bell Labs, Lucent. 1996 - 1996
Post-Doctoral Member, MSRI, UC Berkeley. 1998 – 1998

**Prior Service as an Expert in Cases in Which I Testified at Trial or in a Deposition or Submitted a Report**

Harper v. Moore, N.C. Sup. Ct. No. 21-cv-500085 (2021)
Harper v. Lewis, N.C. Sup. Ct. No. 19-cv-012667 (2019)
Common Cause v. Lewis, N.C. Sup. Ct No. 18-cvs-014001 (2019)
Common Cause v. Rucho, No. 16-CV-1026 (M.D.N.C.) (2017)
Diamond v. Torres, No. 17-CV-5054 (E.D. Pa.)

**Awards and Honors**

Indiana University Distinguished Lecturer, 2024
SIAM Annual Meeting Plenary Speaker 2024
Simons Fellow. Visiting NYU, EPFL, & Leipzig 2024-2025.
Simons Visiting Professor. SLMath. Fall 2023.
IE Block Community Lecture. SIAM. July 2021.
School of Mathematics/ Member. Institute for Advanced Study. 2021.
Defenders of Democracy. National Common Cause. 2018
Fellow of the American Mathematical Society American Mathematical Society. 2015
Simons Visiting Professor. MSRI. Fall 2015
Institute of Mathematical Statistics Fellow. Institute of Mathematical Statistics. June 2012
Faculty Early Career Development (CAREER) Program. National Science Foundation. 2005
Sloan Research Fellowship-Mathematics. Alfred P. Sloan Foundation. 2005
Presidential Early Career Awards for Scientists and Engineers. National Science Foundation. 2005.
School of Mathematics/ Members. Institute for Advanced Study. 2002

**Service to Profession**

Trustee of the American Mathematics Society. 2024 – 2029
ICM2026 Satellite Organizing Committee 2023-2026
Member - Committee on the Profession. AMS. 2024
Co-organizer. Algorithms, Fairness, and Equity Program. MSRI/SLMath. August 2023.
Chair of Scientific Advisory Committee. Pacific Institute of Mathematical Sciences. 2023-Present
Scientific Advisory Committee. Pacific Institute of Mathematical Sciences. 2022- Present
Board of Trustees. ICERM. NSF Math Institute. Brown University. 2017-2023
Member. Committee on Science Policy. SIAM. 2022-Present
Organizing Committee. SIAM online PDE seminar. SIAM. 2022-Present
Participant. Faculty Curriculum on Anti-Racism. Duke Office of Faculty Advancement, 2021

Committee to Select the Gibbs Lecturer for 2020 and 2021. AMS. February 2019
Scientific Advisory Board. Southeast Center for Mathematics and Biology. GA Tech. 2019.
Co-Organizer. Quantifying Gerrymandering. SAMSI. October 2018
Co-Organizer. Regional Gerrymandering Conference. November 2017
Co-Organizer. Interacting particle systems with applications in Biology, Ecology, & Statistics Physics. Duke. 2017.
PDE and Infinite Dimensional Stochastic. Organizer Special Term. MSRI, Berkeley CA. August 2015.
Associate editor Nonlinearity. December 2013
Organized invited session at SPA2013. August 2013
Co-Organizer (with Amarjit Budhiraja at UNC): Seminar on Stochastic Processes 2013. Duke. March 2013
Local Organizer (with Rick Durrett): Woman in Probability III. October 2012
Associate Editor: Journal of Stochastic Partial Differential Equations. September 2012
SAMSI Stochastic Dynamics tradition workshop. November 2010
MFO week-long school on ergodic theory. October 2010
SAMSI Opening Workshop for Stochastic Dynamics. August 2009
Scientific Advisory Committee. NimBios, NSF Math/Bio center. University of Tennessee. 2007-2010
AMS Short Course Selection Committee. 2009
Organizer SAMSI year on stochastic dynamics. 2009
Committee on Committees. AMS. 2008
Organizer Special Term Dynamical Systems. MSRI, Berkeley CA. 2007

**Service to Duke**

Distinguished Professor Selection Committee. University. 2021 - 2024.
A&S Finance Dean Hiring Committee. University. 2019.
Library Committee. University. 2018 - 2024.

**Duke Outreach**

Co-organizer of TriCAMS 2023 at Duke and NCCU (Regional Organizing Committee 2022-Present)
Co-PI Duke RTG in Probability, Analysis, & PDE 2020-2025
Data+ Project Leader. Data+. 2021 2017, 2016
Bass Connections Team Leader. Gerrymandering and the Extent of Democracy in America. North Carolina. 2018.
Organizer. Discovering Research in Mathematics for High School Students. 2011-2023
PRUV Undergraduate research mentor, 2005, 2010, 2011, 2019

**Selected Presentations and Appearances**

Analysis and PDE Seminar: Optimal enhanced dissipation and mixing for a time-periodic, Lipschitz velocity field on 2D Torus. Applied Mathematics Seminar. Stanford University. November 2023.

Random Switching in Fluid Models. Fluid Equations, A Paradigm for Complexity: Regularity vs Blow-up, Deterministic vs Stochastic. BIRS Math Institute. Banff, CA. October 2023.

Analysis and PDE Seminar: Optimal enhanced dissipation and mixing for a time-periodic, Lipschitz velocity field on 2D Torus. PDE and Anlayisis Seminar. UC Berkeley Mathematics. October 2023.

Mathematics Department Colloquium: Computational and Theoretical Challenges in hearing the will of the people in the Vote. UC Berkeley Deprtment Colloquium . September 2023.

A randomly split model for Euler Dynamics. Stochastic PDEs and Related Topics. Brin Mathematics Research Center. University of Maryland. October 2022.

A model for random Euler and Navier Stokes equations based on a randomized splitting of the dynamics.. Small Scale Dynamics in Fluid Motion. Simons Center for Geometry and Physics. June 13, 2022.

Random Splitting for Euler's equations. Mathematical physics at coffee, the first 50 years. University of Geneva. June 2022.

Thinking about the Ergodicity of SPDEs. Unifying concepts in PDEs with randomness. Centre de Rechearch Menthematique. Montreal. May 16, 2022

Using Computational Sampling to Quantify Gerrymandering. Boeing Seminar. Applied Math. University of Washington. March 10, 2022.

A new model of randomly forced Fluid equations. Members seminar. IAS. December 2021.

The Mathematics and Policy of Gerrymandering. IAS. December 2021.

A new model of randomly forced Fluid equations. Princeton Fluids Seminar. November 2021.

Barton Lectures in Computational Mathematics. UNCG. November 2021.

Panel on Qunatifying Gerrymandering. Redistricting and American Democracy . Democracy in America. October 2021.

A new model of randomly forced Fluid equations. Conference on Wave Turbulence. ICEM. October 2021.

Hearing the Will of the People. Joint Stats Meeting. ISM. August 2021.

Non-reversible samplers for Gerrymandering. Non reversible MCMC. Netherlands. August 2021.

IE Block Community Lecture . SIAM. SIAM Annual Meeting. July 2021.

The Gaussian Structure of the Stochastically Forced Burgers Equation. SPDE & Friends. Berlin. May 2021.

Gaussian Structure of Stochastic Burgers. Wave Turbulence online seminar. February 2021.

Gaussian Structure of Burgers Equation. India (online). January 2021.

Sampling to Understand Gerrymandering and Influence Public Policy. Policymaking in Operations Research . MIT. January 2021.

New Sampling Methods of Quantifying Gerrymandering. Brown Applied Math Colloquium. October 2020.

Quantifying and Understanding Gerrymandering - How a quest to understand his state's political geography led a mathematician to court. Public Lecture. ICERM . October 2020.

New Sampling Methods to Quantify Gerrymandering. Redistricting Conference 2020. Duke Law and TRIPODS. IID. March 2020

Quantifying Gerrymandering Using Sampling. AAAS. National Meeting in Seattle. January 2020.

AMS Regional Meeting Plenary Speaker. AMS. Gainesville . 2019.

Long-Time Numerical Simulation of SDEs. SciCADE2019 . Innsbruck. 2019.

Interactions between noise and instabilities.. Stochastic Dynamics Out of Equilibrium. IHP, Paris. July 2018.

Quantifying Gerrymandering: A Mathematician Goes to Court. PCMI Public Lecture. July 2018.

Anatomy of an ergodic theorem. Recent trends in continuous and discrete. Summer School. June 2018.

Quantifying Gerrymandering: A mathematician goes to court. 2018 Niven Lecture. UBC. May 2018.

Ergodicity of Singular SPDEs. Transport and localization in random media: theory and applications. Columbia. May 2018

Approximate/exact controllability and ergodicity for (additive noise) SPDEs/SODEs. Stochastic Partial Differential Equations. CIRM, Marseilles. May 2018.

Discovering the geopolitical structure of the United States through Markov Chain Monte Carlo sampling. Data-driven modeling of complex systems. The Alan Turing Institute, UK. May 2018.

Ergodic and global solutions for singular SPDEs. Frontier Probability Days. Corvallis, Oregon. March 2018.

Quantifying Gerrymandering: a mathematician goes to court. de Leeuw Distinguished Lecture Series. Stanford Mathematics Department. March 2018.

Drawing the line in redistricting (A mathematician's take). Institute for International Studies. Stanford University. March 2018.

A mathematician Goes to Court. 2017 Fields Medal Symposium. October 2017.

Stabilization of Stochastic Dynamics. Turbulence, Mixing, and stability. IPAM. UCLA. January 2017.

Stochastic PDEs. Gene Golub SIAM Summer School. July 2016.

Stabilization and noise. Department Colloquium. Berekey Mathematics Department. November 12, 2015

Stochastic PDEs. Analysis of PDEs of Fluid Mechanics. October 2015

Ergodicity Finite and Infinite dimentional Markov Chains. CRM-PIMS Summer School in Probability. McGill University. July 2015

Dynamics Days 2014. Atlanta GA. January 4, 2014

Stabilization by Noise. November 19, 2013

Uniqueness of the inviscid limit in a simple model damped/driven system.. Probability and Mathematical Physics Seminar. November 5, 2013

Stochastic stabilization of OEDs.. Applied Math Seminar, NYU. September 6, 2013

Stochastic partial differential equations. SPA2013. August 1, 2013

Stabilization by noise. University of Maryland. May 1, 2013

Stabilization by Noise. Conférence en l'honneur d'Etienne Pardoux, CIRM, Marseillais France.. February 14, 2013

Perspectives on Ergodicity. Conference on SPDEs, IMA, Minnesota. January 14, 2013

A Numerical Method for the SDEs from Chemical Equations. Probability and Biology section, 2012 Canadian Mathematical Society (winter meeting). December 1, 2012

Minerva Lectures: Erodicity of Markov Processes: From Chains to SDEs to SPDEs. Mathematics Department, Columbia University. November 1, 2012

Stochastic Stabilization. Inria - Sophia Antipolis. July 1, 2012

A Menagerie of Stabilization. Joint Probability and Analysis Seminar, Nice, France. July 1, 2012

Building Lyapunov Functions (4 lectures). EPSRC Symposium Workshop – Easter Probability Meeting. March 1, 2012

Noise Induced Stability. MBI. February 1, 2012

A Menagerie of Stochastic Stabilization. CAMP/Probability Seminar, University of Chicago. October 18, 2011

A menagerie of stochastic stabilization. Equadiff 2011, Loughborough University. August 1, 2011

Coarse–graining of many–body systems: analysis, computations and applications. July 1, 2011

Ergodicity of systems with singular interaction terms. Stochastic Dynamics Transition Workshop, SAMSI. November 18, 2010

Oberwolfach Seminar: The Ergodic Theory of Markov Processes. Oberwolfach, Germany. October 1, 2010

Malliavin Calculus to prove ergodic theorems for SPDEs. ICM Satellite Conference on Probability and Stochastic Processes Indian Statistical Institute, Bangalore. August 13, 2010

SPDE scaling limits of an Markov chain Montecarlo algorithm. Stochastic Partial Differential Equations: Approximation, Asymptotics and Computation, Newton Institute. June 28, 2010

The spread of randomness. German-American Frontiers of Science, Potsdam Germany. June 1, 2010

How to prove an ergodic theorem. oberwolfach. May 1, 2010

Coupling at infinity. Seminar on Stochastic Processes. March 30, 2010

Long time stochastic simiulation. Imperial College. March 15, 2010

Spectral Gaps in Wasserstien Distance. Ergodic Theory Seminary, Princeton Mathematics. March 4, 2010

Trouble with a chain of stochastic oscillators. PACM, Princeton University. March 2, 2010

Hypo-ellipticity for SPDEs. SPDE program, Newton Institute. March 1, 2010

Numerics of SDEs. Warwick University, UK. February 24, 2010

Long-Time Behavior of Stochastically Forced PDEs.. AMS Joint Meeting, San Francisco. January 14, 2010

Ellipticity and Hypo-ellipticity for SPDEs *or* What is ellipticity in infinite dimensions anyway?. Stochastic Partial Differential Equations, Newton Institute. January 8, 2010

SPDE Limits of the Random Walk Metropolis Algorithm in High Dimensions. SIAM PDE Meeting. December 7, 2009

Stochastic fluctuations in biochemical networks. MBI: Mathematical Developments Arising from Biology. November 9, 2009

What makes infinite-dimensional Markov processes different ?. Stochastic Process and Applications, Berlin. July 1, 2009

Introduction to Ergodicity in Infinite Dimensions. TU Berlin. July 1, 2009

Stochastically forced fluid equations: Transfer between scales and ergodicity.. AMS Sectional Meeting (invited talk). April 4, 2009

Trouble with a chain of stochastic oscillators. PACM. Princeton University. April 3, 2009

What makes the ergodic theory for Markov Chains in infinite dimensions different (and difficult)?. Princeton Ergodic Theory Seminar. March 3, 2009

Ergodicity, Energy Transfer, and Stochastic Partial Differential Equations. Columbia University. Columbia University. December 15, 2008

The Spread of Randomness: Ergodicity in Infinite Dimensions. Mathematisches Forschungsinstitut Oberwolfach. December 15, 2008

The spread of randomness through dimensions. IPAM. November 1, 2008

The spread of randomness through dimensions. IPAM- Mathematical Frontiers in Network Multi-Resolution Analysis. November 1, 2008

Troubles with oscillators. Stanford: JBK85, Workshop on Applied Mathematics In Honor Of Joseph B. Keller. October 1, 2008

What is different about the ergodic theory of stochastic PDEs (vs ODEs). UC Irvine, PDE and Probability Seminar. October 1, 2008

Trouble with a chain of stochastic oscillators. Stochastic Seminar, GaTech. September 1, 2008

Troubles with oscillators. East Midlands Stochastic Analysis Seminars. August 1, 2008

Troubles with chains of anharmonic oscillators. Statistical Mechaniques working group. June 1, 2008

The spread of randomness in infinite dimensions and ergodicity for SPDEs. Stochastic Analysis, Random Fields and Applications, Asscona IT. June 1, 2008

Ergodicity of Degenerately forced SPDEs. Séminaire de Probabilités,Laboratoire de Probabilités et Modèles Aléatoires des Universités Pierre et Marie Curie et Denis Diderot. May 27, 2008

Ergodicity of Degenerately forced SPDEs. ETH, Zurich. May 1, 2008